THEORY OF ROTATION FOR VENUS

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Abstract

By describing the rotational motion of a rigid ellipsoidal body under the influence of the gravitation of the Earth and the inverse-square Solar field, the theory of rotation for the planet Venus is developed. It is possible for Venus to rotate with period of 243.160 days retrograde which is locked into $\frac{T_{\oplus+\text{☽}}}{5T_{\text{♀}}-4T_{\oplus+\text{☽}}}$, where $T_{\oplus+\text{☽}}$ and $T_{\text{♀}}$ are orbital periods of the Earth-Moon system and Venus, resonance of its revolution bout the Sun. It is demonstrated that the rotational motion of Venus is stabilized with its two ends of the axis of minimum moment of inertia pointed toward the Sun and Earth at every inferior conjunction; there is an oscillation within the upper and the lower limit of the resonance locked rotation.

By radar, Goldstein[1] and Pettengill[2] have observed that the rotation of the planet Venus is retrograde with a sidereal period of about -250 days. Carpenter[3] has refined this value to -244 days. The orbital period of the Earth and Venus are 365.257 days and 224.700 days. The observed value of -244 days forced us to speculate that there might be a resonance lock for Venus at $\dfrac{T_{\oplus+D}}{5T_{♀}-4T_{\oplus+D}}$ of its orbital period, or -243.160 days. This seems plausible because the second-harmonic term in the planetary potential will have fore-and-aft symmetry and the axis of minimum moment of inertia of Venus will always line up with the position vector from the Earth to the Sun at every inferior conjunction. Goldreich and Peale[4] have already raised the question of such a possibility. We were asking the computer to check the results of our work before we were aware of theirs.

To investigate the Sun-Earth resonance lock effect on the rotational motion of Venus, we use the methods already developed by Liu and O'Keefe [5,6] for the case of the planet Mercury. If A < B < C are the principal moments of inertia at time t, and if C is taken perpendicular to the orbital plane, then the potential energy of the planet Venus is

$$V = -\frac{G M_{\odot} M_{♀}}{r_{♀}} - \frac{G M_{\odot}[A+B+C-3I(\phi_1)]}{2r_{♀}^3} - \frac{G M_{\oplus+D} M_{♀}}{\rho} - \frac{G M_{\oplus+D}[A+B+C-3I(\phi_2)]}{2\rho^3} \tag{1}$$

where G is the gravitational constant: $M_{\odot}$, $M_{\oplus+D}$ and $M_{♀}$ are the mass of the Sun, the Earth-Moon system and Venus; $r_{♀}$ and ρ are distances from Venus to the Sun and the Earth-Moon system. $I(\phi_1)$ and $I(\phi_2)$ are the moments of inertia around the radius vectors $r_♀$ and ρ,

$$I(\phi_1) = A\cos^2\phi_1 + B\sin^2\phi_1$$
$$I(\phi_2) = A\cos^2\phi_2 + B\sin^2\phi_2$$

in which ϕ_1 and ϕ_2 are the angular displacements of the principal axis, A, in the counterclockwise direction as seen from the north from the position vectors $r_♀$ and ρ respectively.

The Lagrangian of the orbital and rotational motion of Venus is then

$$L = \frac{1}{2}M_♀\left[\left(\frac{dr_♀}{dt}\right)^2 + r_♀^2\left(\frac{df_♀}{dt}\right)^2\right] + \frac{1}{2}C\left[\frac{d(f_♀+\phi_1)}{dt}\right]^2 - V \qquad (2)$$

where $f_♀$ is the true anomaly of Venus.

In the case under consideration, the Lagrange equation of the second kind takes the form

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\phi}_1} - \frac{\partial L}{\partial \phi_1} = 0 \qquad (3)$$

Therefore the rotational motion of Venus is governed by

$$\frac{d}{dt}\left[C\frac{d(f_♀+\phi_1)}{dt}\right] + \frac{3GM_⊙M_♀}{2r_♀^3}(B-A)\sin 2\phi_1$$
$$- \frac{3GM_{⊕+D}M_♀}{4\rho^3}(B-A)\frac{d}{d\phi_1}\cos 2\phi_2 = 0 \qquad (4)$$

The mass ratio of the Sun to the Earth-Moon system is 328390. If the orbits of Venus and the Earth are approximated by Bode's law and if A, B and C are constants, equation (4) becomes

$$\frac{d^2\phi_1}{df^2} + \frac{3\lambda}{2k^2}\left\{\sin 2\phi_1 - \frac{\Psi}{a(1-b\cos f)^{\frac{3}{2}}}\right\} = 0 \tag{5}$$

where

$$a = \frac{2M_\odot}{M_{\oplus+\rightmoon}}\left(\frac{r_\venus^2 + r_{\oplus+\rightmoon}^2}{r_\venus^2}\right)^{\frac{3}{2}} = 3482585$$

$$b = \frac{2\,r_\venus r_{\oplus+\rightmoon}}{r_\venus^2 + r_{\oplus+\rightmoon}^2} = 0.9395883$$

$$k = \frac{T_{\oplus+\rightmoon} - T_\venus}{T_{\oplus+\rightmoon}} = 0.3848167$$

$$f = k f_\venus$$

$$\lambda = \frac{B-A}{C}$$

and

$$\Psi = \frac{d}{d\phi_1}\left[\frac{2r_{\oplus+\rightmoon}^2\cos(2\phi_1+2f) - 2r_{\oplus+\rightmoon} r_\venus \cos(2\phi_1+f) + 2r_\venus^2 \sin(2\phi_1+f)\sin f}{(r_{\oplus+\rightmoon}^2 + r_\venus^2)(1-b\cos f)}\right]$$

in which $r_{\oplus+\rightmoon}$ is the distance from the Earth-Moon system to the Sun.

Equation (5) is very well suited to machine solution. We must, of course, first specify initial conditions. Choosing $\phi_1 = 0$ when inferior conjunction occurs at $f = 0$, solutions of $\Phi_1 = f_\venus + \phi_1$ are generated for various combinations of oblateness parameter λ and initial conditions of $\frac{d\phi_1}{df}$. By repeated numerical integration of equation (5) we find that the rotation of Venus is locked into $\frac{T_{\oplus+\rightmoon}}{5T_\venus - 4T_{\oplus+\rightmoon}}$ resonance of its orbital period with oscillations within an upper and a lower limit. The results of the physical oscillations of Venus will appear elsewhere in detail.

The substance of the theory is that such a weak resonance lock does in fact exist. The physical oscillation of Venus is in resonance ----"oscillation" because of the gravitational torques of the Sun and the Earth and "in resonance" because the maximum gravity gradients are adjusted to come at the same stage of each inferior conjunction. Since the condition of the "Sun-Earth gravitation resonance lock" is due to the angle $\Phi - \left(\frac{5T_{\text{♀}} - 4T_{\oplus+\text{☽}}}{T_{\oplus+\text{☽}}}\right) f_{\text{♀}}$ about $\Phi - \left(\frac{5T_{\text{♀}} - 4T_{\oplus+\text{☽}}}{T_{\oplus+\text{☽}}}\right) f_{\text{♀}} = 0$, the rotation of Venus can remain locked in only when the spin rates are within the range between the upper and the lower limit. It is found that the operation of such a resonance lock depends on the spin rate of Venus. If Venus were given a spin rate higher than that corresponding to the lower limit of its rotational period, the so-called "Sun-Earth gravitational resonance lock" would be broken and the value of $\Phi - \left(\frac{5T_{\text{♀}} - 4T_{\oplus+\text{☽}}}{T_{\oplus+\text{☽}}}\right) f_{\text{♀}}$ would increase without limit. If it were given a spin rate smaller than that corresponding to the upper limit of its rotation period, then the rotation of Venus could not reach the so-called "Sun-Earth gravitation resonance lock" and the value of $\Phi - \left(\frac{5T_{\text{♀}} - 4T_{\oplus+\text{☽}}}{T_{\oplus+\text{☽}}}\right) f_{\text{♀}}$ would decrease monotonicly.

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References and Notes

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